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Mass:

$$\underbrace{\frac{dm}{dt}}_{\substack{\text{Rate of increase of mass}\\ \text{in an open system.}}} = \underbrace{\sum_i \dot{m}_i}_{\substack{\text{Mass flow rate}\\ \text{into the system.}}} - \underbrace{\sum_e \dot{m}_e}_{\substack{\text{Mass flow rate}\\ \text{out of the system.}}} \qquad \left[\frac{\text{kg}}{\text{s}}\right]; \tag{1}$$

$$\text{where,}\quad \dot{m} = \rho A V = \frac{AV}{v} \qquad \left[\frac{\text{kg}}{\text{K}}\right]$$

Energy:

$$\underbrace{\frac{dE}{dt}}_{\substack{\text{Rate of increase}\\ \text{of stored energy}\\ \text{of the open system.}}} = \underbrace{\sum_i \dot{m}_i j_i}_{\substack{\text{Energy transported}\\ \text{by mass flow in.}}} - \underbrace{\sum_e \dot{m}_e j_e}_{\substack{\text{Energy transported}\\ \text{by mass flow out.}}} + \underbrace{\dot{Q}}_{\substack{\text{Rate of heat}\\ \text{transfer into}\\ \text{the system.}}} - \underbrace{\dot{W}_{\text{ext}}}_{\substack{\text{Rate of external}\\ \text{work transfer}\\ \text{out of the system.}}} \qquad [\text{kW}] \tag{2}$$

$$\text{where,}\quad j = h + \text{ke} + \text{pe} = h + \frac{V^2}{2000} + \frac{gz}{1000} \quad \left[\frac{\text{kJ}}{\text{kg}}\right]$$

$$\text{and } \dot{W}_{\text{ext}} = \dot{W}_{\text{sh}} + \dot{W}_{\text{el}} + \dot{W}_B; \quad \dot{W}_{\text{sh}} = 2\pi \frac{N}{60} T; \quad \dot{W}_{\text{el}} = \frac{VI}{1000} \quad [\text{kW}]$$

$$W_B = \int p\,d\cancel{V} \quad [\text{kJ}]; \; \dot{W}_B = \lim_{\Delta t \to 0} \frac{W_B}{\Delta t}; \quad [\text{kW}]$$

Entropy:

$$\underbrace{\frac{dS}{dt}}_{\substack{\text{Rate of increase}\\ \text{of entropy for}\\ \text{an open system.}}} = \underbrace{\sum_i \dot{m}_i s_i}_{\substack{\text{Entropy}\\ \text{transported by}\\ \text{mass flow in.}}} - \underbrace{\sum_e \dot{m}_e s_e}_{\substack{\text{Entropy}\\ \text{transported by}\\ \text{mass flow out.}}} + \underbrace{\frac{\dot{Q}}{T_B}}_{\substack{\text{Entropy}\\ \text{transferred}\\ \text{by heat.}}} + \underbrace{\dot{S}_{\text{gen}}}_{\substack{\text{Rate of}\\ \text{generation of}\\ \text{entropy inside}\\ \text{the system}\\ \text{boundary.}}} \quad \left[\frac{\text{kW}}{\text{K}}\right] \tag{3}$$

$$\text{where, according to the second law,}\quad \dot{S}_{\text{gen}} \geq 0$$

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Closed Steady Systems (Wall, Light bulb, Laptop adapter, Gear box, closed cycles)

Mass Equation: $m = \text{constant}$ (1)

Energy Equation: $0 = \dot{Q} - \dot{W}_{\text{ext}}$ $[\text{kW}]$ where, $\dot{W}_{\text{ext}} = \dot{W}_B + \dot{W}_{\text{sh}} + \dot{W}_{\text{el}}$ (2)

Entropy Equation: $0 = \dfrac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}}$ $\left[\dfrac{\text{kW}}{\text{K}}\right]$; Second law asserts: $\dot{S}_{\text{gen}} \ge 0$ (3)

Single-Flow Open-Steady Systems (pumps, turbines, nozzles, valves, pipes, etc.)

Mass: $\dot{m}_e = \dot{m}_i = \dot{m}$ $\left[\dfrac{\text{kg}}{\text{s}}\right]$ (1)

Energy: $\dot{m}j_e = \dot{m}j_i + \dot{Q} - \dot{W}_{\text{ext}}$ $[\text{kW}]$ (2)

where, $j = h + \text{ke} + \text{pe} = h + \dfrac{V^2}{2000} + \dfrac{gz}{1000}$ $[\text{kJ/kg}]$, $\dot{W}_{\text{ext}} = \dot{W}_B + \dot{W}_{\text{sh}} + \dot{W}_{\text{el}}$ $[\text{kW}]$

Entropy: $\dot{m}s_e = \dot{m}s_i + \dfrac{\dot{Q}}{T_B} + \dot{S}_{\text{gen}}$ $\left[\dfrac{\text{kW}}{\text{K}}\right]$ where, by second law, $\dot{S}_{\text{gen}} \ge 0$ (3)

Closed Processes (Heating water in a tank, piston-cylinder compression)

Mass: $m = \text{constant}$ $[\text{kg}]$ (1)

Energy: $\Delta E = E_f - E_b = Q - W_{\text{ext}}$ or, $m\Delta e = Q - W_{\text{ext}}$ $[\text{kJ}]$ (2)

where, $e = u + \text{ke} + \text{pe} = u + \dfrac{V^2}{2000} + \dfrac{gz}{1000}$ $[\text{kJ/kg}]$; $W_{\text{ext}} = W_B + W_{\text{sh}} + W_{\text{el}}$ $[\text{kJ}]$

Entropy: $\Delta S = S_f - S_b = Q/T_B + S_{\text{gen}}$ or, $m\Delta s = Q/T_B + S_{\text{gen}}$ $[\text{kJ/K}]$ where, $S_{\text{gen}} \ge 0$ (3)

Open Processes (Filling an evacuated tank, filling a propane cylinder, discharge from a tank)

Mass: $\Delta m = m_f - m_b = m_i - m_e$ $[\text{kg}]$ (1)

Energy: $\Delta E = E_f - E_b = m_i j_i - m_e j_e + Q - W_{\text{ext}}$ $[\text{kJ}]$

where, $e = u + \text{ke} + \text{pe} = u + \dfrac{V^2}{2000} + \dfrac{gz}{1000}$, $j = h + \text{ke} + \text{pe} = h + \dfrac{V^2}{2000} + \dfrac{gz}{1000}$ $[\text{kJ/kg}]$ (2)

and $W_{\text{ext}} = W_B + W_{\text{sh}} + W_{\text{el}}$ $[\text{kJ}]$

Entropy: $\Delta S = S_f - S_b = m_i s_i - m_e s_e + Q/T_B + S_{\text{gen}}$ where, $S_{\text{gen}} \ge 0$ (3)

Manual State Evaluation
thermofluids.net>Tables

General State Related Equations: **(applies to any substance)**

$$m = \rho \cancel{V}\,;\ \rho = \frac{1}{v};\ \mathrm{ke} = \frac{V^2}{2000};\ \mathrm{pe} = \frac{gz}{1000};\ \ e \equiv u + ke + pe;\ j \equiv h + ke + pe;\ h \equiv u + pv \qquad (1)$$

$$E = me;;\ \ S = ms;\ KE = m(\mathrm{ke});\ PE = m(\mathrm{pe}) \qquad (2)$$

$$\dot{m} = \rho AV\,;\ \dot{\cancel{V}} = AV\,;\ \dot{E} = \dot{m}e;\ \ \dot{S} = \dot{m}s; \qquad (3)$$

$$Tds = du + pdv = dh - vdp;\ \ c_v \equiv \left(\partial u / \partial T\right)_v;\ c_p \equiv \left(\partial h / \partial T\right)_p \qquad (4)$$

SL Model: (Assumptions: $\rho = \text{constant}\ c_v$ =constant: see Tables>Table-A)

$$\Delta u \equiv u_2 - u_1 = c(T_2 - T_1); \qquad c_v = c_p = c; \qquad (5)$$

$$\Delta h \equiv h_2 - h_1 = \Delta(u + pv) = \Delta u + \Delta(pv) = c(T_2 - T_1) + v(p_2 - p_1) \qquad (6)$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} \qquad (7)$$

PG Model: (Assumptions: $p = \rho RT\,;\ c_v$ =constant: see Tables>Table-C)

$$p = \rho RT = \frac{RT}{v} = \frac{m}{\cancel{V}}RT = \frac{m}{\cancel{V}}\frac{\bar{R}}{\bar{M}}T = \frac{m}{\bar{M}}\bar{R}\frac{T}{\cancel{V}} = n\bar{R}\frac{T}{\cancel{V}}, \qquad \text{where}\ \ R \equiv \frac{\bar{R}}{\bar{M}} \qquad (8)$$

$$\Delta u \equiv u_2 - u_1 = c_v(T_2 - T_1),\ \ \Delta h \equiv h_2 - h_1 = c_p(T_2 - T_1), \quad \text{where} \quad c_p = (c_v + R) \qquad (9)$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1};\ \ \Delta s = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1};\ \text{also,}\ k \equiv \frac{c_p}{c_v},\ c_p = \frac{kR}{k-1};\ \ \text{and}\ \ c_v = \frac{R}{k-1} \qquad (10)$$

For isentropic process: $\dfrac{p_2}{p_1} = \left(\dfrac{\rho_2}{\rho_1}\right)^k = \left(\dfrac{T_2}{T_1}\right)^{\frac{k}{k-1}}$ $\qquad \dfrac{p_2}{p_1} = \left(\dfrac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\dfrac{\rho_2}{\rho_1}\right)^k = \left(\dfrac{v_1}{v_2}\right)^k = \left(\dfrac{\cancel{V}_1}{\cancel{V}_2}\right)^k$ (11)

For polytropic process replace k with n

IG Model: (Assumptions: $p = \rho RT\,;\ c_v$ is function of T : see Tables>Table-D)

$$p = \rho RT = \frac{RT}{v} = \frac{m}{\cancel{V}}RT = \frac{m}{\cancel{V}}\frac{\bar{R}}{\bar{M}}T = \frac{m}{\bar{M}}\bar{R}\frac{T}{\cancel{V}} = n\bar{R}\frac{T}{\cancel{V}} \qquad (12)$$

$$h = h(T),\ u = u(T)\ \ s = s(p,T)\ \ \text{(use ideal gas tables)};\ \ c_p = c_v + R \qquad (13)$$

The temperature dependent part of entropy is separated from the pressure dependent part:

$$\Delta s = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R\ln\frac{p_2}{p_1} = s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1},\ \text{where}\ \ s^0(T)\ \ \text{is tabulated against}\ T\,. \quad (14)$$

PC Model: (see Tables>Table-B) Determine the phase, L, V or M, of the fluid. For vapor use superheated Table. For mixture, use saturation table (if the quality is not known, your goal should be to evaluate the quality first which is the key to finding all specific properties of a mixture). For liquid use the **CL sub-model**.

CL Sub-Model: v, u and s depend on T only. Therefore, use the temperature-sorted saturation table to obtain $v = v_{f@T}$, $u = u_{f@T}$ or $s = s_{f@T}$. To find h, use $h = u + pv = u_{f@T} + pv_{f@T}$.

RG Model: (see Tables>Table-E) $p = Z(p_r, T_r)\rho RT$ where Z, the compressibility factor, is obtained from a chart. p_r and T_r are pressure and temperature normalized by the corresponding critical properties. Just like entropy in the PG or IG model, h and u also have two parts, one temperature dependent and another pressure dependent, in the RG model. The departure of these values from the corresponding IG values are tabulated in the enthalpy and entropy departure charts as functions of p_r and T_r. Therefore, the complete state can be evaluated if p_r and T_r are given.