

Primjer J: Deformacije ravnog nosača metodom analogne grede

Zadano: $F, a, M = F \cdot a, EI_y = \text{konst.}$

J)

Resolucije u osloncima:

- $\sum F_A \Rightarrow F_A + F_B + 2F = 0$
- $\sum M_B \Rightarrow F_A \cdot 2e + M - M + F \cdot e - F \cdot e = 0$

$F_A = 0, F_B = -2F$

$M_A = M = F \cdot e = M_D, M_{B,L} = M - F \cdot e = 0,$
 $M_{B,D} = M_{B,L} - M = -F \cdot e, M_E = 0$

Opterećenje analogne grede:

$$F_1^* = F_2^* = \frac{F \cdot e^2}{EI_y}, F_3^* = F_4^* = \frac{1}{2} \cdot \frac{F \cdot e^2}{EI_y}$$

Resolucije analogne grede:

$\overline{AB}^* : 1. \sum F_A^* = 0 \Rightarrow F_A^* + F_B^* = F_1^* + F_2^* + F_3^* + F_4^*$

$$2. \sum M_A = 0 \Rightarrow 2F_1^* \cdot e + F_3^* \cdot \frac{7}{3}e = F_B^* \cdot 3e \quad | : 3e$$

$$F_B^* = \frac{F \cdot e^2}{3EI_y} \left(2 \cdot 1 + \frac{1}{2} \cdot \frac{7}{3} \right) = \frac{19}{18} \cdot \frac{F \cdot e^2}{EI_y}$$

$$F_A^* = 2F_1^* + F_3^* - F_B^* = \frac{F \cdot e^2}{EI_y} \left(2 + \frac{1}{2} - \frac{19}{18} \right) = \frac{13}{9} \cdot \frac{F \cdot e^2}{EI_y}$$

$\overline{BE}^* : F_E^* = F_B^* - F_4^* = \frac{F \cdot e^2}{EI_y} \left(\frac{19}{18} - \frac{1}{2} \right) = \frac{5}{9} \cdot \frac{F \cdot e^2}{EI_y}$

$$M_E^* = -F_B^* \cdot e + F_4^* \cdot \frac{2}{3}e = \frac{F \cdot e^3}{EI_y} \left(-\frac{19}{18} \cdot 1 + \frac{1}{2} \cdot \frac{2}{3} \right) = -\frac{13}{18} \cdot \frac{F \cdot e^3}{EI_y}$$

Način tangente na elastičnim linijama:

$$\alpha_A = -\alpha_A^* = -F_A^* = -\frac{13}{9} \cdot \frac{F \cdot e^2}{EI_y}, \alpha_B = -\alpha_B^* = F_B^* = \frac{19}{18} \cdot \frac{F \cdot e^2}{EI_y}, \alpha_E = -\alpha_E^* = F_E^* = \frac{5}{9} \cdot \frac{F \cdot e^2}{EI_y}$$

$\alpha_C = -\alpha_C^* = -F_A^* + F_1^* = \frac{F \cdot e^2}{EI_y} \left(-\frac{13}{9} + 1 \right) = -\frac{4}{9} \cdot \frac{F \cdot e^2}{EI_y}, \alpha_D = -\alpha_D^* = -F_A^* + F_1^* + F_2^* = \frac{F \cdot e^2}{EI_y} \left(-\frac{13}{9} + 2 \right) = \frac{5}{9} \cdot \frac{F \cdot e^2}{EI_y}$

Progib grede: $w_A = 0, w_B = 0$

$$w_C = F_A^* \cdot e - F_1^* \cdot \frac{e}{2} = \frac{F \cdot e^3}{EI_y} \left(\frac{13}{9} \cdot 1 - 1 \cdot \frac{1}{2} \right) = \frac{17}{18} \cdot \frac{F \cdot e^3}{EI_y}, w_D = M_D^* \cdot F_B^* \cdot e - F_3^* \cdot \frac{e}{3} = \frac{F \cdot e^3}{EI_y} \left(\frac{19}{18} \cdot 1 - \frac{1}{2} \cdot \frac{1}{3} \right) = \frac{16}{18} \cdot \frac{F \cdot e^3}{EI_y}$$

$$w_E = M_E^* = -\frac{13}{18} \cdot \frac{F \cdot e^3}{EI_y}$$

$F^* = F_A^* = \frac{13}{9} \cdot \frac{F \cdot e^2}{EI_y} = x \cdot \frac{F \cdot e}{EI_y} \Rightarrow x = \frac{13}{9}e \approx 1,4444e$

$$w(x) = F_A^* \cdot x - F \cdot \frac{x}{2} = \frac{F \cdot e^3}{EI_y} \cdot \frac{13}{9} \cdot \frac{13}{9} \cdot \frac{1}{2} = \frac{169}{162} \cdot \frac{F \cdot e^3}{EI_y} \approx 1,04321 \cdot \frac{F \cdot e^3}{EI_y} = w_{\text{max}}$$

(U skorijoj budućnosti, svi primjeri analognih greda biti će iscrtani i ispisani uobičajenom tehnikom, a sada se ovdje daju skenirani iz radnog materijala!).