

**Primjer H: Deformacije ravnog nosača metodom analogne grede**

**Zadano:**  $F, a, M = F \cdot a, EI_y = \text{konst.}$

H)

**Reakcije originalne grede:**

$$1. \sum F_y = 0 \quad F_A + F_B + \frac{5}{2}F = 0$$

$$2. \sum M_A = 0 \quad F \cdot e - \frac{3}{2}F \cdot 2e - M - F_B \cdot 3e = 0 \quad | :3e$$

$$F_B = -F, \quad F_A = -\frac{5}{2}F - F_B = -\frac{3}{2}F$$

$$M_A = -F \cdot e, \quad M_E = F_B \cdot e - M = -M, \quad M_B = -M = -F \cdot e$$

**Optencije grede analogne grede:**

$$F_1^* = \frac{1}{2} \cdot \frac{F \cdot e^2}{EI_y} = F_3^*, \quad F_2^* = \frac{F \cdot e^2}{EI_y}, \quad F^* = \frac{1}{4} \cdot \frac{F \cdot e^2}{EI_y}$$

**Reakcije analogne grede:**

$$AB^*: \quad 1. \sum F_z^* = 0 \quad F_A^* + F_B^* = F_2^* + F_3^*$$

$$2. \sum M_A^* = 0 \quad F_B^* \cdot 3e = F_2^* \cdot \frac{2}{3}e + F_3^* \cdot \frac{4}{3}e \quad | :3e$$

$$F_B^* = \frac{F \cdot e^2}{3EI_y} \left( 1 \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{4}{3} \right) = \frac{2}{3} \cdot \frac{F \cdot e^2}{EI_y}$$

$$F_A^* = F_2^* + F_3^* - F_B^* = \frac{F \cdot e^2}{EI_y} \left( 1 + \frac{1}{2} - \frac{2}{3} \right) = \frac{5}{6} \cdot \frac{F \cdot e^2}{EI_y}$$

$$AC^*: \quad F_C^* = F_1^* + F_A^* = \frac{F \cdot e^2}{EI_y} \left( \frac{1}{2} + \frac{5}{6} \right) = \frac{4}{3} \cdot \frac{F \cdot e^2}{EI_y}$$

$$MC^* = F_1^* \cdot \frac{2}{3}e + F_A^* \cdot e = \frac{F \cdot e^3}{EI_y} \left( \frac{1}{2} \cdot \frac{2}{3} + \frac{5}{6} \cdot 1 \right) = \frac{7}{6} \cdot \frac{F \cdot e^3}{EI_y}$$

**Nagibi tangente na elastičnu liniju:  $\alpha_i = -Q_i^*$**

$$\alpha_A = -Q_A^* = F_A^* = \frac{5}{6} \cdot \frac{F \cdot e^2}{EI_y} \quad \alpha_B = -Q_B^* = -F_B^* = -\frac{2}{3} \cdot \frac{F \cdot e^2}{EI_y} \quad \alpha_C = -Q_C^* = F_C^* = \frac{4}{3} \cdot \frac{F \cdot e^2}{EI_y}$$

$$\alpha_D = -Q_D^* = -F_2^* + F_3^* = \frac{F \cdot e^2}{EI_y} \left( -\frac{2}{3} + \frac{1}{2} + \frac{1}{2} \right) = \frac{1}{12} \cdot \frac{F \cdot e^2}{EI_y} \quad \alpha_E = -Q_E^* = -F_B^* + F_3^* = \frac{F \cdot e^2}{EI_y} \left( -\frac{2}{3} + \frac{1}{2} \right) = -\frac{1}{6} \cdot \frac{F \cdot e^2}{EI_y}$$

**Nagibi grede:  $w_i = M_i^*$**

$$w_C = M_C^* = \frac{7}{6} \cdot \frac{F \cdot e^3}{EI_y} \quad w_D = M_D^* = -F_B^* \cdot 2e + F_3^* \cdot \frac{5}{3}e + F^* \cdot \frac{e}{3} = \frac{F \cdot e^3}{EI_y} \left( -\frac{2}{3} \cdot 2 + \frac{1}{2} \cdot \frac{5}{3} + \frac{1}{4} \cdot \frac{1}{3} \right) = -\frac{5}{12} \cdot \frac{F \cdot e^3}{EI_y}$$

$$w_E = M_E^* = -F_B^* \cdot e + F_3^* \cdot \frac{2}{3}e = \frac{F \cdot e^3}{EI_y} \left( -\frac{2}{3} \cdot 1 + \frac{1}{2} \cdot \frac{2}{3} \right) = -\frac{1}{3} \cdot \frac{F \cdot e^3}{EI_y}$$

**Elastična linija:**

$$y = \frac{1}{2} \cdot \frac{x'}{a} = \frac{x'}{2a} = \frac{x' \cdot F \cdot e}{2a \cdot EI_y} = \frac{1}{2} x' \cdot y' = \frac{1}{2} \cdot \frac{x'^2}{2e} = \frac{x'^2}{4e} = Q_E^* = \frac{1}{6}e \quad \Rightarrow x'^2 = \frac{4}{6}e^2 = \frac{2}{3}e^2$$

$$x' = \frac{2e}{\sqrt{3}} \approx 0,8165 \cdot e$$

$$x_m = 3e - x' \approx 2,1835 \cdot e$$

$$w(x_m) = F^* \cdot \frac{x'}{3} + F_3^* \left( \frac{2}{3}e + x' \right) - F_B^* \cdot (e + x') = \frac{F \cdot e^3}{EI_y} \left( \frac{1}{8} \cdot \frac{2}{3} + \frac{1}{2} \cdot \left( \frac{2}{3} + \frac{2}{\sqrt{3}} \right) - \frac{2}{3} \cdot \left( 1 + \frac{2}{\sqrt{3}} \right) \right) \approx -0,424055 \cdot \frac{F \cdot e^3}{EI_y}$$

(U skorijoj budućnosti, svi primjeri analognih greda bit će iscrtani i ispisani uobičajenom tehnikom, a sada se ovdje daju skenirani iz radnog materijala!)