

**Primjer E: Deformacije ravnog nosača metodom analogne grede**

**Zadano:**  $F, a, M = F \cdot a, EI_y = \text{konst.}$

E)

Reakcije u osovinsima:

$$1. \sum F_z \Rightarrow F_A + F_B + \frac{3}{2}F = 0$$

$$2. \sum M_A \Rightarrow F \cdot a + w - \frac{F}{2} \cdot a - F_B \cdot 3a = 0 \quad | :3a$$

$$F_B = \frac{F}{3} \left( 1 + 1 - \frac{1}{2} \right) = \frac{F}{2}, \quad F_A = -\frac{3}{2}F - F_B = -2F$$

$M_A = -Fa, M_B = w = Fa, M_D = -F \cdot 2a + F_A \cdot a = 0, M_C = 0$

Opterećenje analogne grede:

$$F_1^* = F_2^* = \frac{1}{2} \cdot \frac{F a^2}{EI_y}, \quad F_3^* = \frac{F a^2}{EI_y}, \quad F^* = \frac{1}{4} \cdot \frac{F a^2}{EI_y}$$

Reakcije analogne grede:

$$AB^*: \sum F_z^* \Rightarrow F_A^* - F_B^* - F_2^* + F_3^* = 0$$

$$2. \sum M_A^* \Rightarrow F_2^* \cdot \frac{a}{3} - F_3^* \cdot \frac{2}{3}a + F_B^* \cdot 3a = 0 \quad | :3a$$

$$F_B^* = \frac{F a^2}{3EI_y} \left( -\frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} \right) = \frac{13}{18} \frac{F a^2}{EI_y}$$

$$F_A^* = F_B^* + F_2^* - F_3^* = \frac{F a^2}{EI_y} \left( \frac{13}{18} + \frac{1}{2} - 1 \right) = \frac{4}{18} \frac{F a^2}{EI_y} = \frac{2}{9} \frac{F a^2}{EI_y}$$

$$AC^*: F_C^* = F_1^* + F_A^* = \frac{F a^2}{EI_y} \left( \frac{1}{2} + \frac{4}{18} \right) = \frac{13}{18} \frac{F a^2}{EI_y}$$

$$M_C^* = F_1^* \cdot \frac{2}{3}a + F_A^* \cdot a = \frac{F a^3}{EI_y} \left( \frac{1}{2} \cdot \frac{2}{3} + \frac{4}{18} \cdot 1 \right) = \frac{10}{18} \frac{F a^3}{EI_y}$$

Nagibi: tangente na elastičnu liniju:

$$\alpha_A = -\alpha_A^* = F_A^* = \frac{2}{9} \frac{F a^2}{EI_y}, \quad \alpha_B = -\alpha_B^* = F_B^* = \frac{13}{18} \frac{F a^2}{EI_y}, \quad \alpha_E = -\alpha_E^* = F_C^* = \frac{13}{18} \frac{F a^2}{EI_y}$$

$$\alpha_D = -\alpha_D^* = F_A^* - F_2^* = \frac{F a^2}{EI_y} \left( \frac{4}{18} - \frac{1}{2} \right) = -\frac{5}{18} \frac{F a^2}{EI_y}, \quad \alpha_E = -\alpha_E^* = F_A^* - F_2^* + F_3^* = \frac{F a^2}{EI_y} \left( \frac{4}{18} - \frac{1}{2} + \frac{1}{4} \right) = -\frac{1}{36} \frac{F a^2}{EI_y}$$

Progibi grede:  $w_A = w_B = 0$

$$w_C = M_C^* = \frac{5}{9} \frac{F a^3}{EI_y}, \quad w_D = M_D^* = -F_A^* \cdot a + F_2^* \cdot \frac{2}{3}a = \frac{F a^3}{EI_y} \left( -\frac{4}{18} \cdot 1 + \frac{1}{2} \cdot \frac{2}{3} \right) = \frac{1}{9} \frac{F a^3}{EI_y}$$

$$w_E = M_E^* = -F_A^* \cdot 2a + F_2^* \cdot \frac{5}{3}a - F_3^* \cdot \frac{a}{3} = \frac{F a^3}{EI_y} \left( -\frac{2}{9} \cdot 2 + \frac{1}{2} \cdot \frac{5}{3} - \frac{1}{4} \cdot \frac{1}{3} \right) = \frac{11}{36} \frac{F a^3}{EI_y}$$

$$x_{m1} = 1,2548 \cdot a \rightarrow w(x_{m1}) = -0,026918 \frac{F a^3}{EI_y}, \quad x_{m2} = 3,054 \cdot a \rightarrow w(x_{m2}) = 0,306313 \frac{F a^3}{EI_y}$$