

Primjer D: Deformacije ravnog nosača metodom analogne grede

Zadano: $F, a, M = F \cdot a, EI_y = \text{konst.}$

D)

Reakcije u oštrocima:

$$\sum F_z \rightarrow F_A + F_B + 2F \rightarrow$$

$$\sum M_A \rightarrow F \cdot 0 - F \cdot 3a - M - F_B \cdot 2a = -F \cdot 1/2a$$

$$F_B = \frac{F}{2}(-2-1) = -\frac{3}{2}F, F_A = -2F - F_B = -\frac{F}{2}$$

$$M_{A/L} = -F \cdot 0, M_{A/D} = M_{A/L} + M = a, M_B = -F \cdot a, M_C = a$$

Opterećenje analogne grede:

$$F_1^* = \frac{1}{2} \cdot \frac{F \cdot a^2}{EI_y} = \frac{F a^2}{2EI_y}, F_2^* = \frac{F a^2}{EI_y}, F^* = \frac{1}{4} \frac{F a^2}{EI_y}$$

Reakcije analogne grede:

$$\overline{AB}^*: \sum F_z^* \rightarrow F_A^* + F_B^* - F_2^* = 0$$

$$\sum M_B^* \rightarrow F_A^* \cdot 2a - F_2^* \cdot \frac{2}{3}a = -F \cdot \frac{1}{2}a$$

$$F_A^* = \frac{F_2^*}{3} = \frac{1}{3} \cdot \frac{F a^2}{EI_y}, F_B^* = F_2^* - F_A^* = \frac{2}{3} \cdot \frac{F a^2}{EI_y}$$

$$\overline{AC}^*: F_C^* = F_1^* + F_A^* = \frac{F a^2}{EI_y} \left(\frac{1}{2} + \frac{1}{3} \right) = \frac{5}{6} \frac{F a^2}{EI_y}$$

$$M_C^* = F_1^* \cdot \frac{2}{3}a + F_A^* \cdot a = \frac{F a^2}{EI_y} \left(\frac{1}{2} \cdot \frac{2}{3} + \frac{1}{3} \cdot 1 \right) = \frac{2}{3} \frac{F a^2}{EI_y}$$

$$\overline{BE}^*: F_E^* = F_3^* + F_B^* = \frac{F a^2}{EI_y} \left(\frac{1}{2} + \frac{2}{3} \right) = \frac{7}{6} \frac{F a^2}{EI_y}$$

$$M_E^* = F_B^* \cdot a + F_3^* \cdot \frac{2}{3}a = \frac{F a^2}{EI_y} \left(\frac{2}{3} \cdot 1 + \frac{1}{2} \cdot \frac{2}{3} \right) = \frac{F a^2}{EI_y}$$

Napitci tangente na dva t. linija grede:

$$d_A = -Q_A^* = +F_A^* = \frac{1}{3} \cdot \frac{F a^2}{EI_y} \quad d_B = -Q_B^* = -F_B^* = -\frac{2}{3} \cdot \frac{F a^2}{EI_y}$$

$$d_C = -Q_C^* = F_C^* = \frac{5}{6} \cdot \frac{F a^2}{EI_y} \quad d_E = -Q_E^* = -F_E^* = -\frac{7}{6} \cdot \frac{F a^2}{EI_y}$$

Prognje grede: $w_A = w_B = 0$

$$w_C = M_C^* = \frac{2}{3} \cdot \frac{F a^3}{EI_y}, w_E = M_E^* = \frac{F a^3}{EI_y}$$

$$w_D = M_D^* = -F_A^* \cdot a + F^* \cdot \frac{a}{3} = \frac{F a^3}{EI_y} \left(-\frac{1}{3} \cdot 1 + \frac{1}{6} \cdot \frac{1}{3} \right) = -\frac{1}{4} \cdot \frac{F a^3}{EI_y}$$

Mjesto maksimalnog prognja:

$$Q_3^* = F_A^* - F^* = 0 \rightarrow F^* = \frac{1}{2} X_m' \cdot \frac{F a^2}{4a} = \frac{X_m'^2}{4e} = \frac{1}{3} \cdot \frac{F a^2}{4e} \rightarrow X_m'^2 = \frac{4}{3} e^2 \rightarrow X_m' = \frac{2}{\sqrt{3}} e$$

$$X_m = X_m' + a = 2,1547 \cdot e$$

$$w(X_m) = F_A^* \cdot X_m + F^* \cdot \frac{1}{3} X_m = \frac{F a^3}{EI_y} \left(-\frac{1}{3} \cdot \frac{2}{\sqrt{3}} + \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{\sqrt{3}} \right) = -\frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{\sqrt{3}} \cdot \frac{F a^3}{EI_y} = -\frac{4}{9\sqrt{3}} \cdot \frac{F a^3}{EI_y} \approx -0,2566 \cdot \frac{F a^3}{EI_y}$$

$$X_m = 2,1547 \cdot e$$