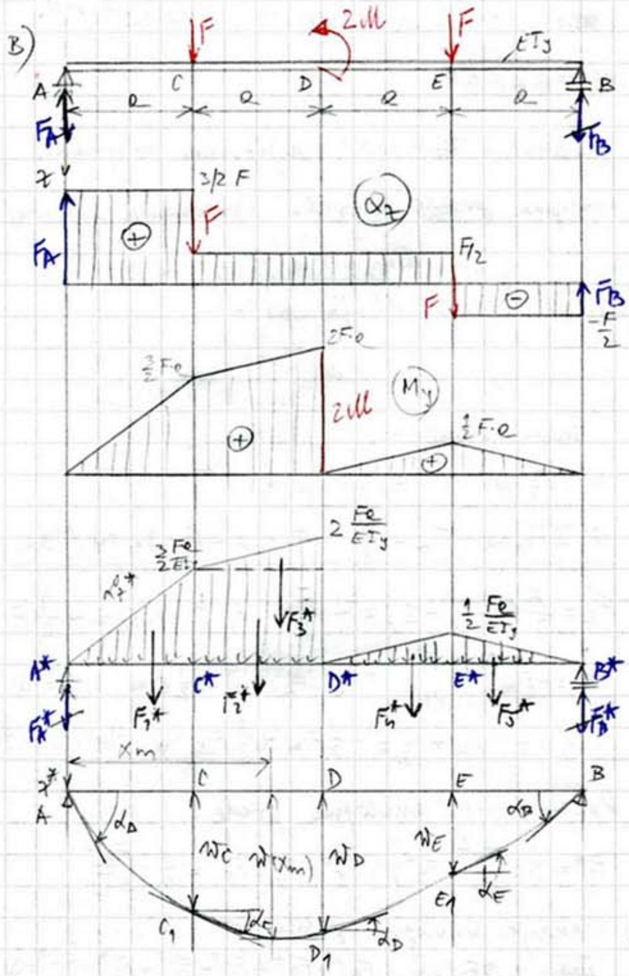
Primjer B: Deformacije ravnog nosača metodom analogne grede

Zadano: F , a , $\mathcal{M} = F \cdot a$, $EI_y = \text{konst.}$



Jedn. ravnoteže:

1. $\Sigma F_z = 0 \quad F_A + F_B + 2F = 0$

2. $\Sigma M_A = 0 \quad 2\mathcal{M} - F\cdot a - F\cdot 3a - F_B \cdot 4a = 0 \quad /:4a$

$F_B = \frac{F}{4}(2-4) = -\frac{F}{2}, \quad F_A = -2F - F_B = -\frac{3}{2}F$

$M_C = F_A \cdot a = \frac{3}{2}F\cdot a, \quad M_E = F_B \cdot a = \frac{1}{2}F\cdot a$

$M_{D,L} = F_A \cdot 2a - F\cdot a = F\cdot a\left(\frac{3}{2}\cdot 2 - 1\right) = 2F\cdot a$

$M_{D,D} = M_{D,L} - 2\mathcal{M} = 2F\cdot a - 2\cdot F\cdot a = 0$

Opterećenje analogne grede:

$F_1^* = \frac{3}{4}\frac{Fa^2}{EI_y}, \quad F_2^* = \frac{3}{2}\cdot\frac{Fa^2}{EI_y}, \quad F_3^* = F_4^* = F_5^* = \frac{1}{4}\frac{Fa^2}{EI_y}$

Reakcije analogne grede:

1. $\Sigma F_z^* = 0 \quad F_A^* + F_B^* + F_1^* + F_2^* + F_3^* + F_4^* + F_5^* = 0$

2. $\Sigma M_A^* = 0 \quad F_1^* \cdot \frac{2}{3}a + F_2^* \cdot \frac{3}{2}a + F_3^* \cdot \frac{5}{3}a + 2\cdot F_4^* \cdot 3a + F_B^* \cdot 4a = 0 \quad /:4a$

$F_B^* = -\frac{Fa^2}{4EI_y}\left(\frac{3}{4}\cdot\frac{2}{3} + \frac{3}{2}\cdot\frac{3}{2} + \frac{1}{4}\cdot\frac{5}{3} + 2\cdot\frac{1}{4}\cdot 3\right) = \frac{-7}{6}\cdot\frac{Fa^2}{EI_y}$

$F_A^* = \frac{Fa^2}{EI_y}\left(\frac{7}{6} - \frac{3}{4} - \frac{3}{2} - 3\cdot\frac{1}{4}\right) = -\frac{11}{6}\cdot\frac{Fa^2}{EI_y}$

Nagibi tangente na elastičnu liniju:

$\alpha_A = -Q_A^* = -F_A^* = -\frac{11}{6}\cdot\frac{Fa^2}{EI_y}$, $\quad \alpha_B = -Q_B^* = F_B^* = \frac{7}{6}\cdot\frac{Fa^2}{EI_y}$,

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$\alpha_C = -Q_C^* = -F_A^* + F_1^* = \frac{Fa^2}{EI_y}\left(-\frac{11}{6} + \frac{3}{4}\right) = -\frac{13}{12}\cdot\frac{Fa^2}{EI_y}$, $\quad \alpha_E = -Q_E^* = F_B^* - F_5^* = \frac{Fa^2}{EI_y}\left(\frac{7}{6} - \frac{1}{4}\right) = \frac{11}{12}\cdot\frac{Fa^2}{EI_y}$

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$\alpha_D = -Q_D^* = F_B^* - 2\cdot F_4^* = \frac{Fa^2}{EI_y}\left(\frac{7}{6} - 2\cdot\frac{1}{4}\right) = \frac{8}{12}\cdot\frac{Fa^2}{EI_y} = \frac{2}{3}\cdot\frac{Fa^2}{EI_y}$

$0{,}66667$

Progibi grede: $w_A = w_B = 0 \qquad x_m = 1{,}6515\cdot a \rightarrow w(x_m) = 1{,}94774\frac{Fa^3}{EI_y}$

$w_C = M_C^* = F_A^* \cdot a - F_1^* \cdot \frac{a}{3} = \frac{Fa^3}{EI_y}\left(\frac{11}{6}\cdot 1 - \frac{3}{4}\cdot\frac{1}{3}\right) = \frac{19}{12}\cdot\frac{Fa^3}{EI_y}$

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$w_D = M_D^* = F_B^* \cdot 2a - 2\cdot F_4^* \cdot a = \frac{Fa^3}{EI_y}\left(\frac{7}{6}\cdot 2 - \cancel{2}\cdot\frac{1}{\cancel{4}_2}\cdot 1\right) = \frac{11}{6}\cdot\frac{Fa^3}{EI_y}$

$1{,}83333$

$w_E = M_E^* = F_B^* \cdot a - F_5^* \cdot \frac{a}{3} = \frac{Fa^3}{EI_y}\left(\frac{7}{6}\cdot 1 - \frac{1}{4}\cdot\frac{1}{3}\right) = \frac{13}{12}\cdot\frac{Fa^3}{EI_y}$

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